The Investment Decision – Accounting for Uncertainty with Sensitivity Analysis

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1.0 Introduction

When making the investment decision, decision makers seek to identify investment. (hereafter referred to as "projects") that will maximise shareholder value. To identify the projects that will add the greatest value, a range of investment appraisal methods can be employed such as the Payback Period, the Internal Rate of Return or the Net Present Value (NPV). Before employing these methods, all cash flow implications of a given project must be forecast and the periodic net cash flows arising from the project be estimated. However, decision makers cannot predict the future and must recognise a degree of uncertainty surrounds all forecasted figures. As such, it is vital that decision makers use the appropriate methods to account for uncertainty in project forecasts, but also recognise the strengths and weaknesses of these methods. The aim of this article is to explain and critically appraise sensitivity analysis as a method to account for uncertainty in the investment appraisal process, specifically when using the NPV method:

2.0 Sensitivity Analysis Explained

To calculate the NPV of a project, any number of variables must be forecast such as unit demand, selling price, material costs, direct labour requirements etc. The forecasts of these variables (hereafter referred to as "inputs") will depend on forecasts of factors such as economic growth, market interest rates, inflation etc., and so there is naturally a degree of uncertainty or margin for error associated with the forecasts of these inputs. To account for this uncertainty, decision makers can employ sensitivity analysis, which involves allowing one project input to vary and examining the effect of this variation on the NPV of the project. The exact manner in which sensitivity analysis is employed will depend on the nature of the input being varied and the type of information being sought, but its defining characteristic is that only one input is allowed to vary at any time. This allows the effect of incorrectly forecasting this input to be observed in isolation, and for the decision maker to gauge the susceptibility of the financial viability of the project to forecasting error associated with this input. To illustrate how sensitivity analysis can be used to gauge the susceptibility of a project's NPV to isolated variations in project inputs, consider the following simple investment opportunity available to a company:

ABC Ltd is considering purchasing a new piece of equipment that will enable it to manufacture and sell a new product. The cost of purchasing and installing the equipment is uncertain, but the Financial Manager forecasts it will cost approximately €4,000,000. The equipment would have a useful economic life of 5 years after which it would be scrapped at a zero net cost. Demand for the product is forecast at 10,000 units per annum and the selling price per unit is forecast at €200 per unit. Variable costs per unit are forecast at €40 per unit, and incremental overheads per annum are forecast at €400,000. Assume a cost of capital of 10% and ignore all tax implications.

Assuming that all forecasts are accurate and using a present value annuity factor (PVAF) of 3.791 to calculate the present value of the cash flows from years 1 to 5, the project has a positive NPV of €549,200 and the Financial Manager would advise the project to be undertaken.

NPV based on forecasts

	Year 0	Years 1 to 5	
Equipment	-€ 4,000,000		
Revenue		€ 2,000,000	(10,000 x €200)
Variable Costs		-€ 400,000	(10,000 x €40)
Overheads		-€ 400,000	
Net Cash Flow	-€ 4,000,000	€ 1,200,000	
PVAF @ 10%	1.000	3.791	
PV	-€ 4,000,000	€ 4,549,200	
NPV	€ 549,200		

However, the actual values of the project inputs may turn out to be higher or lower than forecast, and therefore the project's NPV may turn out to be higher or lower than forecast. The following sections outline two ways in which sensitivity analysis can be used to assess the impact of actual input values deviating from forecasted input values.

2.1 Realistic Deviation

The first method that can be used to implement sensitivity analysis involves identifying a level by which an input's actual value might realistically deviate, both unfavourably and favourably, from its forecasted value, and estimating the range of NPV's a project could have given these deviations. This process can be repeated for all project inputs, and a table of NPV ranges can be created that enables the decision maker to assess the impact of deviation in each output on a project's NPV. To illustrate, consider again the example of ABC Ltd. Assume that having analysed relevant data, the Financial Manager believes actual input values might realistically deviate from forecasted values as follows:

Table 1: Realistic Unfavourable and Favourable Deviations in Inputs

Input	Unfavourable (U)	Forecast	Favourable (F)
Equipment Cost	€4,700,000	€4,000,000	€3,700,000
Demand in Units	9,000*	10,000	12,000
Selling Price per Unit	€190	€200	€215
Variable Cost per Unit	€52	€40	€35
Overheads per Annum	€600,000	€400,000	€300,000**

The Financial Manager would then recalculate the NPV of the project multiple times, where each recalculation involves allowing just one input's value to deviate unfavourably and then favourably. This would be done for each input, resulting in a table of NPVs. For ABC Ltd, this table would look as follows:

Table 2: Range of NPVs given Realistic Deviations in Inputs

Input	NPV U	NPV Forecast	NPV F
Equipment Cost	-€150,800	€549,200	€849,200
Demand in Units	-€57,360*	€549,200	€1,762,320
Selling Price per Unit	€170,100	€549,200	€1,117,850
Variable Cost per Unit	€94,280	€549,200	€738,750
Overheads per Annum	-€209,000	€549,200	€928,300**

For clarity, the recalculations of the project NPV for unfavourable deviation in demand in units and favourable deviation in overheads per annum are as follows:

NPV with unfavourable deviation in demand in units

	Year 0	Years 1 to 5	
Equipment	-€ 4,000,000		
Revenue		€ 1,800,000	(9,000 x €200)
Variable Costs		-€ 360,000	(9,000 x €40)
Overheads		-€ 400,000	
Net Cash Flow	-€ 4,000,000	€ 1,040,000	
PVAF @ 10%	1.000	3.791	
PV	-€ 4,000,000	€ 3,942,640	
NPV	-€ 57,360*		

NPV with favourable deviation in overheads per annum

	Year 0	Years 1 to 5	
Equipment	-€ 4,000,000		
Revenue		€ 2,000,000	(10,000 x €200)
Variable Costs		-€ 400,000	(10,000 x €40)
Overheads		-€ 300,000	
Net Cash Flow	-€ 4,000,000	€ 1,300,000	
PVAF @ 10%	1.000	3.791	
PV	-€ 4,000,000	€ 4,928,300	
NPV	€ 928,300**		

Whilst there is no precise method for interpreting the information in Table 2, there is useful information that can facilitate more informed decision making. For example, the table indicates that financial viability is unlikely to be adversely affected by forecasting error in relation to selling price per unit or variable cost per unit, as realistic unfavourable deviations in both inputs still result in positive NPVs. On the other hand, it appears that forecasting errors with regards to equipment cost and overheads per annum represent the greatest risks to financial viability, as realistic unfavourable deviations in both inputs result in relatively large negative NPVs.

2.2 Margin of Safety

The second method that can be used to implement sensitivity analysis involves identifying the value of an input that results in a project having a zero NPV, and then calculating a margin of safety that represents the percentage by which the actual input value can deviate from the forecasted input value before the project would no longer be financially viable. This process can be repeated for all project inputs, and a list of margins of safety can be prepared and suitably interpreted. Returning to the example of ABC Ltd, trial and error demonstrates that selling price per unit would have to fall to €185.51 for the project to have a zero NPV:

Selling price resulting in zero NPV

	Year 0	Years 1 to 5	
Equipment	-€ 4,000,000		
Revenue		€ 1,855,131	(10,000 x €185.51)
Variable Costs		-€ 400,000	(10,000 x €40)
Overheads		-€ 400,000	
Net Cash Flow	-€ 4,000,000	€ 1,055,131	
PVAF @ 10%	1.000	3.791	
PV	-€ 4,000,000	€ 4,000,000	
NPV	€ 0		

This could then be represented as a percentage margin of safety using the following formula:

$$Margin\ of\ Safety = \left(\frac{Input_{0NPV} - Input_{Forecast}}{Input_{Forecast}}\right) \times 100\%$$

$$\left(\frac{€185.51 - €200}{€200}\right) \times 100\% = -7.25\%$$

This figure of -7.25% indicates that the actual selling price could be 7.25% below the forecasted selling price before the project would have a negative NPV i.e. there is a 7.25% margin of safety in relation to the forecast for selling price. Repeating the process for the other inputs results in the following margins of safety:

Table 3: Margins of Safety

Input	Margin of Safety
Equipment Cost	13.73%
Demand in Units	-9.05%
Selling Price per Unit	-7.25%
Variable Cost per Unit	36.20%
Overheads per Annum	36.22%

Rather than relying entirely on a trial-and-error process, it is also possible to *estimate* the value of an input that would result in a zero NPV. Assuming that the relationship between input and NPV is approximately linear, linear interpolation can be used to determine the margin of safety associated with an input's forecast. In relation to the variable cost per unit in the project under consideration by ABC Ltd, a little trial and error reveals that a variable cost per unit of €60 results in a negative NPV of -€209,000. Using €40 as the variable cost per unit that results in a positive NPV i.e. €549,200, linear interpolation can be applied and a margin of safety estimated as follows:

$$Input_{0NPV} = Input_1 + \frac{(Input_2 - Input_1) \times NPV_1}{NPV_1 - NPV_2}$$

$$€40 + \frac{(€60 - €40) \times €549{,}200}{€549{,}200 - (-€209{,}000)} = €54.49$$

$$\left(\frac{€54.49 - €40}{€40}\right) \times 100\% = 36.23\%$$

If the input is a variable cash flow stream, there is another method that can be used to calculate the margin of safety. This involves representing the project's NPV as a percentage of the present value of the cash flow stream, which is then interpreted as the percentage by which the present value of the cash flow stream would have to change before the project would have a zero NPV. In relation to ABC Ltd, this method could be used to calculate the margin of safety in relation to the overheads per annum as follows:

$$PV_{Overheads} = €400{,}000 \times 3.791 = €1{,}516{,}400$$

$$Margin\ of\ saftey = \frac{NPV}{PV_{Overheads}} \times 100\%$$

$$\frac{€549{,}200}{€1{,}516{,}400} \times 100\% = 36.22\%$$

In the ABC Ltd example, we ignored taxation to simplify the example. However, in practice, if the cash flow is tax deductible, then the present value of the after-tax cash flow stream would have to be calculated. Assuming a tax rate of 15%, this could be calculated as follows for ABC Ltd's overheads:

$$PV_{Overheads\ After\ Tax} = €400{,}000 \times (1 - 0.15) \times 3.791 = €1{,}288{,}940$$

3.0 Benefits and Limitations of Sensitivity Analysis

Like many other tools used to aid financial decision making, sensitivity analysis offers a number of benefits to users, but is also limited in what it can offer. Before using sensitivity analysis, decision makers should be aware of its pros and cons.

3.1 Demonstrates Uncertainty

The NPV method's decision rule is that projects with positive NPVs should be accepted whilst projects with negative NPVs should be rejected. Having factored project risk into the discount rate applied, decision makers can be lulled into a false sense of security that they are applying a black and white rule about which there is no uncertainty or ambiguity. Sensitivity analysis helps to uncover the uncertainty inherent in the calculation of a project's NPV by demonstrating the vulnerability of a project's financial viability to forecasting errors. Decision makers can then apply more rigorous decision rules that factor in this uncertainty e.g. accept projects with a positive NPV whose forecasted inputs have margins of safety greater than X%.

3.2 Identifies Key Inputs

Project appraisal may require forecasting of many inputs which can place significant strain on decision makers time and efforts. Sensitivity analysis helps to identify the inputs whose forecasts have the biggest influence on a project's financial viability, and so can be used to focus decision makers attention and forecasting efforts. For example, in Table 2 above, unfavourable deviations in three inputs result in the project having a negative NPV, and so decision makers may decide to focus their time and effort improving the accuracy of these inputs' forecasts or may attempt to eliminate uncertainty in these inputs via contractual arrangements. Decision makers may even narrow their attention on two of these inputs, namely equipment cost and overheads per annum, as whilst the project's NPV under an unfavourable deviation in demand in units is negative, the magnitude is relatively small, and a favourable deviation in demand in units appears to offer the greatest scope for a better-than-expected outcome i.e. NPV of €1,762,320.

3.3 Subjectively Applied

Although sensitivity analysis enables an enhanced decision-making system to be applied, the manner in which this system is applied is determined by the decision maker. In this regard, sensitivity analysis is inherently subjective. For example, assume the acceptance or rejection of the project under consideration by ABC Ltd depends on each input's margin of safety exceeding a minimum acceptable level. If that level was set at 7%, the project would be accepted, as all margins of safety exceed this level. On the other hand, if the level was set at 8%, the margin of safety for selling price would not meet the threshold and the project may be rejected. As there is no generally accepted method for determining minimum acceptable margins of safety, whether the project is accepted or not will likely depend on the personal preferences of one or more individuals. When making financial decisions, it is desirable to have a system in place that would result in the same decision being made regardless of who the decision maker is. Sensitivity analysis does not provide this.

3.4 Unrealistic View of Risk

The generation of tables of NPV ranges or margins of safety enable decision makers to visualise the risk underlying the calculation of a project's NPV. However, by only allowing for deviation in one input at a time, this visualisation is unlikely to be realistic. In reality, the factors that cause an input's actual value to deviate from its forecast value will not affect that input in isolation but will affect multiple inputs simultaneously. For example, if ABC Ltd's forecasts were made in 2021, it would likely have incorrectly accounted for future global supply chain conditions and the interest rate environment when making its forecasts. As such, not only might actual demand for its products have deviated from forecast demand, but so too might actual selling price, variable costs and overheads. Now, assuming ABC had set a minimum acceptable margin of safety of 5%, the project would have been accepted. However, what ABC would have failed to account for is that a 4% simultaneous unfavourable deviation in demand, selling price, variable costs and overheads would have resulted in a negative NPV of -€103,459. By failing to recognise that inputs can and will deviate simultaneously, sensitivity analysis provides decision makers with an unrealistic view of risk.

4% unfavourable deviation in four inputs

	Year 0	Years 1 to 5		
Equipment	-€ 4,000,000			
Revenue		€	1,843,200	(9,600 x €192)
Variable Costs		-€	399,360	(9,600 x €41.6)
Overheads		-€	416,000	
Net Cash Flow	-€ 4,000,000	€	1,027,840	
PVAF @ 10%	1.000	3.791		
PV	-€ 4,000,000	€	3,896,541	
NPV	-€ 103,459			